Paper Reference(s)

**6665/01**

# Edexcel GCE

## Core Mathematics C3
## Advanced Level

## Thursday 14 June 2007 – Afternoon
## Time: 1 hour 30 minutes

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae (Green) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

### Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

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**1.** Find the exact solutions to the equations

(*a*) $\ln x + \ln 3 = \ln 6$,

**(2)**

(*b*) $e^x + 3e^{-x} = 4$.

**(4)**

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**2.**

$$f(x) = \frac{2x+3}{x+2} - \frac{9+2x}{2x^2+3x-2}, \quad x > \tfrac{1}{2}.$$

(*a*) Show that $f(x) = \dfrac{4x-6}{2x-1}$.

**(7)**

(*b*) Hence, or otherwise, find $f'(x)$ in its simplest form.

**(3)**

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**3.** A curve $C$ has equation $y = x^2e^x$.

(*a*) Find $\dfrac{dy}{dx}$, using the product rule for differentiation.

**(3)**

(*b*) Hence find the coordinates of the turning points of $C$.

**(3)**

(*c*) Find $\dfrac{d^2y}{dx^2}$.

**(2)**

(*d*) Determine the nature of each turning point of the curve $C$.

**(2)**

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**4.** $$f(x) = -x^3 + 3x^2 - 1.$$

(*a*) Show that the equation f(*x*) = 0 can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}.$$

**(2)**

(*b*) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of $x_2$, $x_3$ and $x_4$, giving all your answers to 4 decimal places.

**(2)**

(*c*) Show that $x = 0.653$ is a root of f(*x*) = 0 correct to 3 decimal places.

**(3)**

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**5.** The functions f and g are defined by

$$f : \mapsto \ln(2x - 1), \qquad x \in \mathbb{R},\ x > \tfrac{1}{2},$$

$$g : \mapsto \frac{2}{x-3}, \qquad x \in \mathbb{R},\ x \neq 3.$$

(*a*) Find the exact value of fg(4).

**(2)**

(*b*) Find the inverse function $f^{-1}(x)$, stating its domain.

**(4)**

(*c*) Sketch the graph of $y = |g(x)|$. Indicate clearly the equation of the vertical asymptote and the coordinates of the point at which the graph crosses the *y*-axis.

**(3)**

(*d*) Find the exact values of *x* for which $\left|\frac{2}{x-3}\right| = 3$.

**(3)**

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**6.** (*a*) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

**(4)**

(*b*) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$.

**(2)**

(*c*) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places.

**(5)**

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**7.** (*a*) Prove that

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2 \operatorname{cosec} 2\theta, \quad \theta \neq 90n°.$$

**(4)**

(*b*) Sketch the graph of $y = 2 \operatorname{cosec} 2\theta$ for $0° < \theta < 360°$.

**(2)**

(*c*) Solve, for $0° < \theta < 360°$, the equation

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 3$$

giving your answers to 1 decimal place.

**(6)**

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**8.** The amount of a certain type of drug in the bloodstream $t$ hours after it has been taken is given by the formula

$$x = De^{-\frac{1}{8}t},$$

where $x$ is the amount of the drug in the bloodstream in milligrams and $D$ is the dose given in milligrams.

A dose of 10 mg of the drug is given.

(*a*) Find the amount of the drug in the bloodstream 5 hours after the dose is given.
Give your answer in mg to 3 decimal places.

**(2)**

A second dose of 10 mg is given after 5 hours.

(*b*) Show that the amount of the drug in the bloodstream 1 hour after the second dose is 13.549 mg to 3 decimal places.

**(2)**

No more doses of the drug are given. At time $T$ hours after the second dose is given, the amount of the drug in the bloodstream is 3 mg.

(*c*) Find the value of $T$.

**(3)**

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**TOTAL FOR PAPER: 75 MARKS**

**END**

# June 2007
# 6665 Core Mathematics C3
# Mark Scheme

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** (*a*) | $\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$ | M1 |
| | $x = 2$ (only this answer) | A1 (cso) (2) |
| (*b*) | $(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form) | M1 |
| | $(e^x - 3)(e^x - 1) = 0$ | |
| | $e^x = 3$ or $e^x = 1$ Solving quadratic | M1 dep |
| | $x = \ln 3$, $x = 0$ (or ln 1) | M1 A1 (4) |
| | | **(6 marks)** |

Notes: (a) Answer $x = 2$ with no working or no incorrect working seen: M1A1

Note: $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

(b) 1st M1 for attempting to multiply through by $e^x$ : Allow *y*, *X* , even *x*, for $e^x$

2nd M1 is for solving quadratic as far as getting two values for $e^x$ or *y* or *X* etc

3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)

A1 is for ln3 **and** ln1 or 0 (Both required and no further solutions)

| | | |
|---|---|---|
| **2.** (*a*) | $2x^2 + 3x - 2 = (2x - 1)(x + 2)$ at any stage<br>$\mathrm{f}(x) = \dfrac{(2x+3)(2x-1)-(9+2x)}{(2x-1)(x+2)}$ f.t. on error in denominator factors<br>(need not be single fraction)<br>Simplifying numerator to quadratic form ⊦<br>Correct **numerator** $= \dfrac{4x^2+2x-12}{[(2x-1)(x+2)]}$<br>Factorising numerator, **with** a denominator $= \dfrac{2(2x-3)(x+2)}{(2x-1)(x+2)}$ o.e.<br>$= \dfrac{4x-6}{2x-1}$ (✱) | B1<br>M1, A1√<br>M1<br>A1<br>M1<br>A1 cso (7) |
| Alt.(*a*) | $2x^2 + 3x - 2 = (2x - 1)(x + 2)$ at any stage B1<br>$\mathrm{f}(x) = \dfrac{(2x+3)(2x^2+3x-2)-(9+2x)(x+2)}{(x+2)(2x^2+3x-2)}$ M1A1 f.t.<br>$= \dfrac{4x^3+10x^2-8x-24}{(x+2)(2x^2+3x-2)}$<br>$= \dfrac{2(x+2)(2x^2+x-6)}{(x+2)(2x^2+3x-2)}$ or $\dfrac{2(2x-3)(x^2+4x+4)}{(x+2)(2x^2+3x+2)}$ o.e.<br>Any one linear factor × quadratic factor in **numerator** M1, A1<br>$= \dfrac{2(x+2)(x+2)(2x-3)}{(x+2)(2x^2+3x-2)}$ o.e. M1<br>$= \dfrac{2(2x-3)}{2x-1}$ $\dfrac{4x-6}{2x-1}$ (✱) A1 | |
| (*b*) | Complete method for $\mathrm{f}'(x)$; e.g $\mathrm{f}'(x) = \dfrac{(2x-1)\times 4-(4x-6)\times 2}{(2x-1)^2}$ o.e<br>$= \dfrac{8}{(2x-1)^2}$ or $8(2x - 1)^{-2}$<br>Not treating $\mathrm{f}^{-1}$ (for $\mathrm{f}'$) as misread | M1 A1<br>A1 (3)<br>**(10 marks)** |

Notes: (a) 1st M1 in either version is for correct method

1st A1 Allow $\dfrac{2x+3(2x-1)-(9+2x)}{(2x-1)(x+2)}$ or $\dfrac{(2x+3)(2x-1)-9+2x}{(2x-1)(x+2)}$ or $\dfrac{2x+3(2x-1)-9+2x}{(2x-1)(x+2)}$ ( fractions)

2nd M1 in (main a) is for forming 3 term quadratic in **numerator**

3rd M1 is for factorising their quadratic (usual rules) ; factor of 2 need not be extracted

(✱) A1 is given answer so is cso

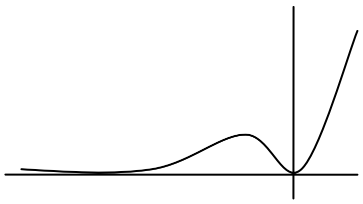
Alt :(a) 3rd M1 is for factorising resulting quadratic

(b) SC: For M allow ± given expression or one error in product rule

Alt: Attempt at $\mathrm{f}(x) = 2 - 4(2x-1)^{-1}$ and diff. M1; $k(2x-1)^{-2}$ A1; A1 as above

Accept $8(4x^2 - 4x + 1)^{-1}$.

Differentiating original function – mark as scheme.

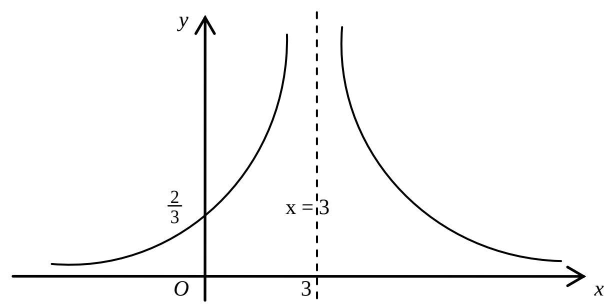
| Question Number | Scheme | Marks |
|---|---|---|
| **3.** (*a*) | $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2\mathrm{e}^x + 2x\mathrm{e}^x$ | M1,A1,A1 (3) |
| (*b*) | If $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, $\mathrm{e}^x(x^2 + 2x) = 0$ setting (*a*) = 0<br>$[\mathrm{e}^x \neq 0]$ $x(x + 2) = 0$<br>$(x = 0)$ $x = -2$<br>$x = 0, y = 0$ **and** $x = -2, y = 4\mathrm{e}^{-2}$ ( = 0.54…) | M1<br><br>A1<br>A1 √ (3) |
| (*c*) | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = x^2\mathrm{e}^x + 2x\mathrm{e}^x + 2x\mathrm{e}^x + 2\mathrm{e}^x$ $\left[= (x^2 + 4x + 2)\mathrm{e}^x\right]$ | M1, A1 (2) |
| (*d*)<br><br><br><br>Alt.(*d*) | $x = 0$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ (=2) $x = -2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$ [ $= -2\mathrm{e}^{-2}$ ( = –0.270…)]<br>M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's $x$ value(s) from (b)<br>$\therefore$ minimum $\therefore$ maximum<br><br>For M1:<br>Evaluate, or state sign of, $\frac{\mathrm{d}y}{\mathrm{d}x}$ at two appropriate values – on either side of at least one of their answers from (b) or<br>Evaluate $y$ at two appropriate values – on either side of at least one of their answers from (b) or<br>Sketch curve<br> | M1<br><br>A1 (cso) (2)<br><br><br><br><br><br><br>**(10 marks)** |

Notes: (a) M for attempt at $f(x)g'(x) + f'(x)g(x)$
1st A1 for one correct, 2nd A1 for the other correct.
**Note that $x^2e^x$ on its own scores no marks**

(b) 1st A1 ($x = 0$) may be omitted, but for
2nd A1 both sets of coordinates needed ; f.t only on candidate's $x = -2$

(c) M1 requires complete method for candidate's (a), result may be unsimplified for A1

(d) A1 is cso; $x = 0$, min, and $x = -2$, max and no incorrect working seen,
or (in alternative) sign of $\frac{dy}{dx}$ either side correct, or values of $y$ appropriate to t.p.
Need only consider the quadratic, as may assume $\mathrm{e}^x > 0$.
**If all marks gained** in (a) and (c), and correct $x$ values, give M1A1 for correct statements with no working

| Question Number | Scheme | Marks |
|---|---|---|
| **4.** (*a*) | $x^2(3-x)-1=0$ o.e. (e.g. $x^2(-x+3)=1$)<br>$x=\sqrt{\dfrac{1}{3-x}}$ (✱)<br>Note(✱), answer is given: need to see appropriate working and A1 is cso<br>[Reverse process: Squaring and non-fractional equation M1, form f($x$) A1] | M1<br>A1 (cso) (2) |
| (*b*) | $x_2 = 0.6455$, $x_3 = 0.6517$, $x_4 = 0.6526$<br>1st B1 is for one correct, 2nd B1 for other two correct<br>If all three are to greater accuracy, award B0 B1 | B1; B1 (2) |
| (*c*) | Choose values in interval (0.6525, 0.6535) or tighter and evaluate both<br>f(0.6525) = –0.0005 ( 372… f(0.6535) = 0.002 (101…<br>At least one correct "up to bracket", i.e. -0.0005 or 0.002<br>**Change of sign,** $\therefore$ $x$ = 0.653 is a root (correct) to 3 d.p.<br>Requires both correct "up to bracket" and conclusion as above | M1<br><br>A1<br>A1 (3)<br><br>**(7 marks)** |
| Alt (i)<br><br><br>Alt (ii) | Continued iterations at least as far as $x_6$ M1<br>$x_5 = 0.65268$, $x_6 = 0.6527$, $x_7 = \ldots$ two correct to at least 4 s.f. A1<br>Conclusion : Two values correct to 4 d.p., so 0.653 is root to 3 d.p. A1<br>If use g(0.6525) = 0.6527..>0.6525 and g(0.6535) = 0.6528..<0.6535 M1A1<br>Conclusion : Both results correct, so 0.653 is root to 3 d.p. A1 | |
| **5.** (*a*) | Finding g(4) = k and f(k) = …. or fg($x$) = $\ln\left(\dfrac{4}{x-3}-1\right)$<br>[ f(2) = ln(2x2 – 1) fg(4) = ln(4 – 1)] = $\ln 3$ | M1<br>A1 (2) |
| (*b*) | $y=\ln(2x-1) \Rightarrow e^y = 2x-1$ or $e^x = 2y-1$<br>$f^{-1}(x) = \frac{1}{2}(e^x+1)$ Allow $y=\frac{1}{2}(e^x+1)$<br>Domain $x \in \Re$ [Allow $\Re$ , all reals, $(-\infty, \infty)$ ] independent | M1, A1<br>A1<br>B1 (4) |
| (*c*) | <br>Shape, and $x$-axis should appear to be asymptote<br>**Equation $x$ = 3 needed,** may see in diagram (ignore others)<br>Intercept (0, $\frac{2}{3}$) no other; accept y = ⅔ (0.67) or on graph | B1<br>B1 ind.<br>B1 ind (3) |
| (*d*)<br><br><br><br>Alt: | $\dfrac{2}{x-3}=3 \Rightarrow x = 3\frac{2}{3}$ or exact equiv.<br>$\dfrac{2}{x-3}=-3$ , $\Rightarrow x = 2\frac{1}{3}$ or exact equiv.<br>Note: $2 = 3(x+3)$ or $2 = 3(-x-3)$ o.e. is M0A0<br>Squaring to quadratic ($9x^2 - 54x + 77 = 0$) **and** solving M1; B1A1 | B1<br>M1, A1 (3)<br><br>**(12 marks)** |

| | | | |
|---|---|---|---|
| **6.** | (*a*) | Complete method for $R$: e.g. $R\cos\alpha = 3$, $R\sin\alpha = 2$, $R = \sqrt{(3^2 + 2^2)}$<br>$R = \sqrt{13}$ or 3.61 (or more accurate)<br>Complete method for $\tan\alpha = \frac{2}{3}$ [Allow $\tan\alpha = \frac{3}{2}$]<br>$\alpha = 0.588$ (Allow 33.7°) | M1<br>A1<br>M1<br>A1 (4) |
| | (*b*) | Greatest value = $\left(\sqrt{13}\right)^4 = 169$ | M1, A1 (2) |
| | (*c*) | $\sin(x + 0.588) = \frac{1}{\sqrt{13}}$ ( = 0.27735…) $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$<br>$(x + 0.588) = 0.281(03\ldots)$ or 16.1°<br>$(x + 0.588) = \pi - 0.28103\ldots$<br>Must be $\pi -$ their 0.281 or 180° – their 16.1°<br>or $(x + 0.588) = 2\pi + 0.28103\ldots$<br>Must be $2\pi +$ their 0.281 or 360° + their 16.1°<br>$x = 2.273$ or $x = 5.976$ (awrt) Both (radians only)<br>If 0.281 or 16.1° not seen, correct answers imply this A mark | M1<br>A1<br>M1<br><br>M1<br><br>A1 (5)<br>**(11 marks)** |

Notes: (a) 1st M1 for correct method for R

2nd M1 for correct method for $\tan\alpha$

No working at all: M1A1 for √13, M1A1 for 0.588 or 33.7°.

N.B. Rcos α = 2, Rsin α = 3 used, can still score M1A1 for R, but loses the A mark for α.

cosα = 3, sin α = 2: apply the same marking.

(b) M1 for realising $\sin(x + \alpha) = \pm 1$, so finding $R^4$.

(c) Working in mixed degrees/rads : first two marks available

Working consistently in degrees: Possible to score first 4 marks

[Degree answers, just for reference only, are 130.2° and 342.4°]

Third M1 can be gained for candidate's 0.281 – candidate's 0.588 + 2π or equiv. in degrees

One of the answers correct in radians or degrees implies the corresponding M mark.

Alt: (c) (i) Squaring to form quadratic in $\sin x$ or $\cos x$ — M1

$[13\cos^2 x - 4\cos x - 8 = 0, \quad 13\sin^2 x - 6\sin x - 3 = 0]$

Correct values for $\cos x = 0.953\ldots$, $-0.646$; or $\sin x = 0.767$, 2.27 awrt — A1

For any one value of $\cos x$ or $\sin x$, correct method for two values of $x$ — M1

$x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere — A1

Checking other values (0.307, 4.011 or 0.869, 3.449) and discarding — M1

(ii) Squaring and forming equation of form $a\cos 2x + b\sin 2x = c$

$$9\sin^2 x + 4\cos^2 x + 12\sin 2x = 1 \Rightarrow 12\sin 2x + 5\cos 2x = 11$$

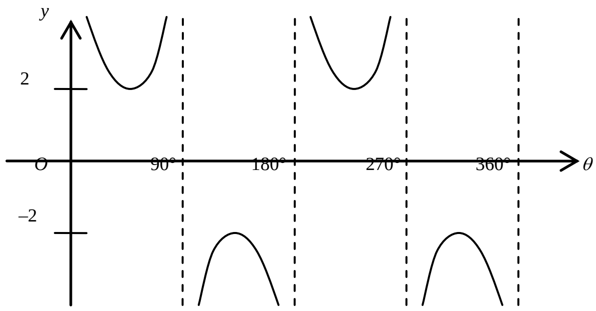
Setting up to solve using R formula e.g. $\sqrt{13}\cos(2x - 1.176) = 11$ — M1

$$(2x - 1.176) = \cos^{-1}\left(\frac{11}{\sqrt{13}}\right) = 0.562(0\ldots \quad (\alpha)$$ — A1

$$(2x - 1.176) = 2\pi - \alpha,\ 2\pi + \alpha, \ldots\ldots\ldots$$ — M1

$x = 2.273$ or $x = 5.976$ (awrt) Both seen anywhere — A1

Checking other values and discarding — M1

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** (*a*) | $\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}$<br>M1 Use of common denominator to obtain single fraction<br>$=\frac{1}{\cos\theta\sin\theta}$<br>M1 Use of appropriate trig identity (in this case $\sin^2\theta+\cos^2\theta=1$)<br>$=\frac{1}{\frac{1}{2}\sin 2\theta}$ Use of $\sin 2\theta=2\sin\theta\cos\theta$<br>$=2\text{cosec}2\theta$ (✱) | M1<br><br>M1<br><br>M1<br>A1 cso (4) |
| Alt.(*a*) | $\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=\tan\theta+\frac{1}{\tan\theta}=\frac{\tan^2\theta+1}{\tan\theta}$ M1<br>$=\frac{\sec^2\theta}{\tan\theta}$ M1<br>$=\frac{1}{\cos\theta\sin\theta}=\frac{1}{\frac{1}{2}\sin 2\theta}$ M1<br>$=2\text{cosec}2\theta$ (✱) (cso) A1<br>If show two expressions are equal, need conclusion such as QED, tick, true. | |
| (*b*) | <br><br>Shape (May be translated but need to see 4"sections")<br>T.P.s at $y=\pm 2$, asymptotic at correct $x$-values (dotted lines not required) | B1<br><br>B1 dep. (2) |
| (*c*) | $2\text{cosec}2\theta=3$<br>$\sin 2\theta=\frac{2}{3}$ Allow $\frac{2}{\sin 2\theta}=3$ [M1 for equation in $\sin 2\theta$]<br>$(2\theta)$ = [ 41.810…°, 138.189…° ; 401.810…°, 498.189…°]<br>1st M1 for $\alpha$, $180-\alpha$ ; 2nd M1 adding 360° to at least one of values<br>$\theta$ = 20.9°, 69.1°, 200.9°, 249.1° (1 d.p.) awrt | M1, A1<br><br>M1; M1 |
| Note | 1st A1 for any two correct, 2nd A1 for other two<br>Extra solutions in range lose final A1 only<br>SC: Final 4 marks: $\theta$ = 20.9°, after M0M0 is B1; record as M0M0A1A0 | A1,A1 (6) |
| Alt.(*c*) | $\tan\theta+\frac{1}{\tan\theta}=3$ and form quadratic, $\tan^2\theta-3\tan\theta+1=0$ M1, A1<br>(M1 for attempt to multiply through by tanθ, A1 for correct equation above)<br>Solving quadratic [ $\tan\theta=\frac{3\pm\sqrt{5}}{2}$ = 2.618… or = 0.3819…] M1<br>$\theta$ = 69.1°, 249.1° $\theta$ = 20.9°, 200.9° (1 d.p.) M1, A1, A1<br>(M1 is for one use of 180° + $\alpha$ °, A1A1 as for main scheme) | **(12 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **8.** (*a*) | $D = 10$, $t = 5$, $x = 10e^{-\frac{1}{8}\times 5}$<br>$= 5.353$ awrt | M1<br>A1 (2) |
| (*b*) | $D = 10 + 10e^{-\frac{5}{8}}$, $t = 1$, $x = 15.3526\ldots \times e^{-\frac{1}{8}}$<br>$x = 13.549$ (✱) | M1<br>A1 cso (2) |
| Alt.(*b*) | $x = 10e^{-\frac{1}{8}\times 6} + 10e^{-\frac{1}{8}\times 1}$ M1 $x = 13.549$ (✱) A1 cso | |
| (*c*) | $15.3526...e^{-\frac{1}{8}T} = 3$<br>$e^{-\frac{1}{8}T} = \dfrac{3}{15.3526...} = 0.1954...$<br>$-\dfrac{1}{8}T = \ln 0.1954...$<br>$T = 13.06\ldots$ or 13.1 or 13 | M1<br><br>M1<br>A1 (3)<br>**(7 marks)** |

Notes: (b) (main scheme) M1 is for $(10 + 10e^{-\frac{5}{8}})e^{-\frac{1}{8}}$, or {10 + their(a)} $e^{-\frac{1}{8}}$

**N.B.** The answer is given. There are many correct answers seen which deserve M0A0 or M1A0

(c) 1st M is for $(10 + 10e^{-\frac{5}{8}})\, e^{-\frac{T}{8}} = 3$ o.e.

2nd M is for converting $e^{-\frac{T}{8}} = k$ (k > 0) to $-\dfrac{T}{8} = \ln k$. This is independent of 1st M.

Trial and improvement: M1 as scheme,
M1 correct process for their equation (two equal to 3 s.f.)
A1 as scheme